Numerical Answers

Magnetism

Problems labelled A are straightforward, those marked B are supposed to be more demanding, those marked C are intended to make students think, and those marked S are synoptic. There are online physics tutorials on WebLearn under Physics for Chemists.

1. Electric current.

Drift speed

1.1A This is in the lecture handout.

1.2A 18.2 mol of electrons, i.e. 1.75 MC.

1.3A From the molar volume $n = 5.86 \times 10^{28}$ m^{-3} (1 electron per atom)
hence $v = 7.94 \times 10^{-7}$ m s^{-1}.

1.4A From the density and rmm $n = 1.46 \times 10^{28}$ m^{-3} (1 ion per formula unit)
hence $v = 4.09 \times 10^{-8}$ m s^{-1}.

1.5A Each ion contributes equally to the current (this assumption is almost correct for RbBr, better than 2SF) hence $i = i_+ + i_- = 2nAve$ and $v = 8.1 \times 10^{-8}$ m s^{-1}.

Resistivity and conductivity.

1.6A $R = V / i = 576\ \Omega$.

1.7A $R = \rho l / A = 0.156\ \Omega$

1.8A Resistivity $\rho = RA / l = 0.64\ \Omega$ m
Conductivity $G = 1/\rho = 1.6$ S m^{-1}

Molar conductivity

1.9B For the standard solution the conductivity is $\kappa = c\Lambda = 12.9\ \text{mS cm}^{-1}$ and the measured conductance is 35.2 mS.

By scaling, the conductivity of the solution is

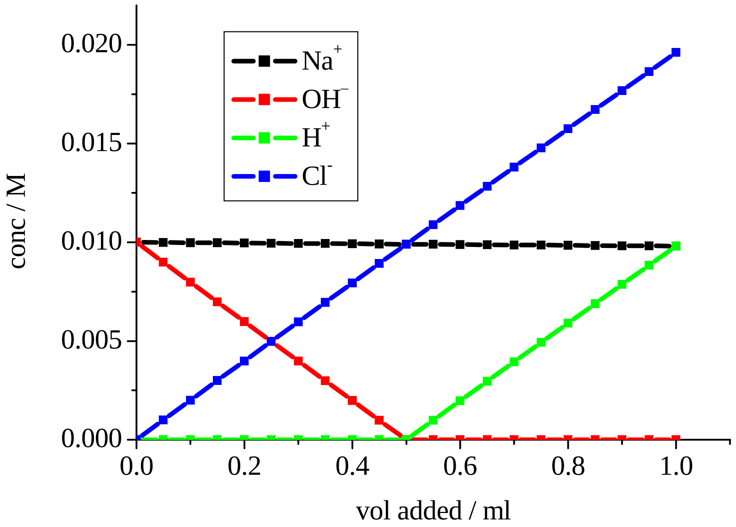
$$\kappa = 12.9\ \text{mS cm}^{-1} \times \frac{31.6\ \text{mS}}{35.2\ \text{mS}} = 11.6\ \text{mS cm}^{-1}$$

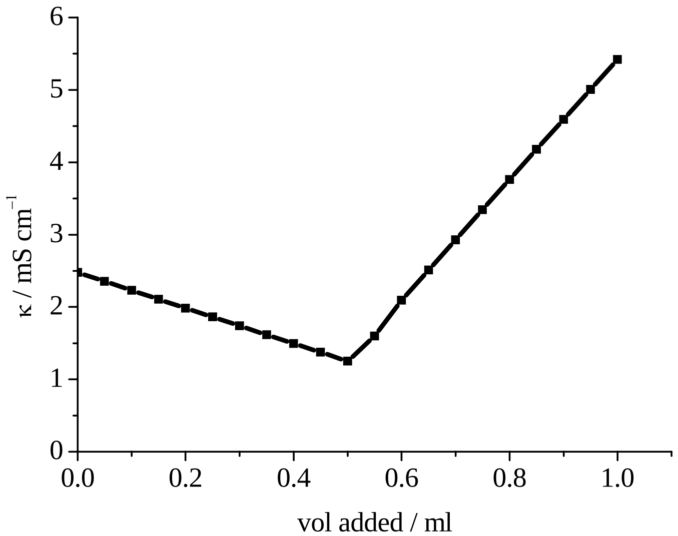
1.10B (i) $\kappa = \sum_i c_i \Lambda_i = 2.48\ \text{mS cm}^{-1}$

(ii) The volume of acid added is $0.5\ \text{cm}^3$, the only important ions present are Na^+ and Cl^- and the concentrations are $0.0100\ \text{M} \times \frac{50.0\ \text{cm}^3}{50.5\ \text{cm}^3} = 0.0099\ \text{M}$.

$\kappa = 1.25\ \text{mS cm}^{-1}$.

(iii) $[Na^+] = [H^+] = 0.0098$ M, $[Cl^-] = 0.0196$ M. $\kappa = 5.42\ \text{mS cm}^{-1}$





κ decreases to the end point because mobile OH^- ions are being replaced by less mobile Cl^- ions. After the end point κ increases because of the addition of ions. The increase is rapid because of the high mobility of the H^+ ions.

1.11B This was done as an example in the lecture in SI units.

$\Lambda_0(\text{HOAc}) = \Lambda_0(\text{H}^+) + \Lambda_0(\text{OAc}^-) = \Lambda_0(\text{HCl}) + \Lambda_0(\text{NaOAc}) - \Lambda_0(\text{NaCl})$

= 390.7 S cm^2 mol^{-1}.

1.12A (a) $\Lambda_0(\text{NaCl}) = \Lambda_0(\text{Na}^+) + \Lambda_0(\text{Cl}^-) = 126.5 \text{ S cm}^2 \text{ mol}^{-1}$

(b) $\kappa = c\Lambda_0 = 4.048 \text{ S cm}^2 \text{ dm}^{-3} = 4.048 \text{ mS cm}^{-1} = 0.4048 \text{ S m}^{-1}$

(c) $u(\text{Na}^+) = 5.20\times10^{-4} \text{ S cm}^2 \text{ C}^{-1} = 5.20\times10^{-8} \text{ S m}^2 \text{ C}^{-1} = 5.20\times10^{-8} \text{ m}^2 \text{ s}^{-1} \text{ V}^{-1}$

$u(\text{Cl}^-) = 7.91\times10^{-4} \text{ S cm}^2 \text{ C}^{-1} = 7.91\times10^{-8} \text{ S m}^2 \text{ C}^{-1} = 7.91\times10^{-8} \text{ m}^2 \text{ s}^{-1} \text{ V}^{-1}$

The hard part is sorting the units out, you can see that the two SI forms are equivalent by using Ohm's law to express S in terms of A and V.

(d) (i) $j = \dfrac{i}{A} = \dfrac{VG}{A} = \dfrac{\kappa V}{l} = 5.18 \text{ mA cm}^{-2} = 51.8 \text{ A m}^{-2}$.

(ii) $v(\text{Na}^+) = 6.65\times10^{-6} \text{ m s}^{-1}$ and $v(\text{Cl}^-) = 1.01\times10^{-5} \text{ m s}^{-1}$.

2. Magnetic force

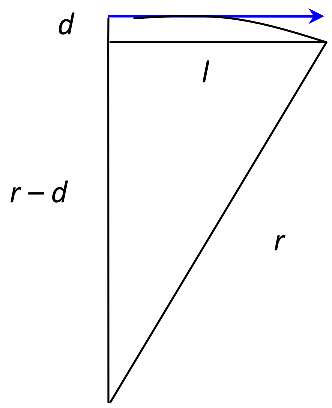
Force on a moving charged particle in a magnetic field

2.1A (i) $\vec{\mathbf{F}} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = -9.01\times10^{-17}\,\vec{\mathbf{j}}$ N

(ii) r = 26.2 mm.

2.2B $m = \dfrac{eB^2r^2}{2V} = 1.30\times10^{-25} \text{ kg} = 78.0 \text{ u}$.

2.3C In the field the electrons perform a circular orbit with radius r (diagram)



The distance to the screen is *l* and the displacement of the beam on the screen is *d*. By Pythagoras $(r-d)^2 + l^2 = r^2$, hence $r = \dfrac{d^2 + l^2}{2d}$

Since *l* = 0.40 m and *d* = 1 mm, *r* = 80 m.
After acceleration through 8 kV the electron velocity is 5.3×10^{7} m s^{-1}, so *B* = 3.8 μT, substantially smaller than the earth's field.

Crossed fields

2.4 $e/m = v^2/2V = 1.76 \times 10^{11}$ C kg^{-1}, which is the same as the accepted value.

2.5 $v = \sqrt{2eV/m} = 8.72 \times 10^{4}$ m s^{-1} and $E/B = v \Rightarrow B = 0.115$ T.

2.6 $n = \dfrac{Bi}{V_H de} = 5.89 \times 10^{28}$ m^{-3}.

Sorting the units out is likely to be a problem. Stick to SI.

2.7 (*a*) The volume of the unit cell is 1.42×10^{-27} m^3, and this contains 4 Rb, 16 Ag and 20 I atoms, hence the number densities are $n(\mathrm{Rb}) = 2.82 \times 10^{27}$ m^{-3}, $n(\mathrm{Ag}) = 1.13 \times 10^{28}$ m^{-3} and $n(\mathrm{I}) = 1.41 \times 10^{28}$ m^{-3}.
(*b*) (i) The magnetic force is $q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = qvB\vec{\mathbf{k}}$. Since the potential observed is positive we conclude that the current carriers are positive ions.
(ii) $n = \dfrac{Bi}{V_H de} = 1.1 \times 10^{28}$ m^{-3}, which implies that the current carriers are most likely to be silver ions.

Magnetic field induced by current

3.1A $\vec{\mathbf{B}} = 3.14\times10^{-6}\vec{\mathbf{k}}$ T.

Magnetic dipoles

3.2A $\mu = \dfrac{el}{2m} = \dfrac{e\hbar}{2m_e}$

3.3A $\Delta E = g\mu_N B = 2.65\times10^{-25}$ J, and the frequency of the transition is therefore $\nu = \Delta E / h = 400$ MHz.

Current induced by magnetic field.

3.3B Lenz's law.

3.4S (*a*) Balancing forces $\dfrac{m_e v^2}{r} = \dfrac{Ze^2}{4\pi\varepsilon_0 r^2} \Rightarrow m_e v^2 = \dfrac{Ze^2}{4\pi\varepsilon_0 r}$

Quantisation of a.m. $m_e vr = n\hbar \Rightarrow v = \dfrac{n\hbar}{m_e r}$

Substituting for *v*: $m_e\left(\dfrac{n\hbar}{m_e r}\right)^2 = \dfrac{Ze^2}{4\pi\varepsilon_0 r} \Rightarrow r = \dfrac{(n\hbar)^2 4\pi\varepsilon_0}{Zm_e e^2}$

and hence $v = \dfrac{Ze^2}{n\hbar 4\pi\varepsilon_0}$

(*b*) kinetic energy: $KE = \dfrac{1}{2}m_e v^2 = \dfrac{Z^2 m_e e^4}{2n^2\hbar^2(4\pi\varepsilon_0)^2}$

potential energy: $PE = -\dfrac{Ze^2}{4\pi\varepsilon_0 r} = -\dfrac{Z^2 m_e e^4}{n^2\hbar^2(4\pi\varepsilon_0)^2}$

hence total energy $E = -\dfrac{Z^2 m_e e^4}{2n^2\hbar^2(4\pi\varepsilon_0)^2}$

(*c*) $B = \dfrac{\mu_0 qv}{4\pi r^2} = \dfrac{\mu_0 Z^3 m_e^2 e^7}{4\pi(n\hbar)^5(4\pi\varepsilon_0)^3}$ (charge is *e*)

(*d*) The radius and speed are the same, but now the circulating charge is *Ze*, hence $B = \dfrac{\mu_0 qv}{4\pi r^2} = \dfrac{\mu_0 Z^4 m_e^2 e^7}{4\pi(n\hbar)^5(4\pi\varepsilon_0)^3}$ = 0.391 T when *n* = 2.

(*e*) The energy difference is found to be $2\mu_B B = \dfrac{\mu_0}{4\pi}\dfrac{m_e e^8}{\hbar^4(4\pi\varepsilon_0)^3}\dfrac{Z^4}{n^5}$, which has the value 7.25×10^{-24} J for *n* = 2. Given the simple nature of the Bohr model the agreement is remarkable (too good to be true).

(*f*) The Bohr and Schrödinger formulas for the energy are identical. The Bohr model gives the correct energy levels for one-electron systems, but breaks down when electron repulsion needs to be taken into account.

(*g*) The combination of fundamental constants predicted by the Bohr model is remarkably correct, including the dependence on Z^4. [This result does not simply follow from dimensional analysis because it is $\frac{Z^4 R\alpha^2}{n^3 l(l+1)}$, where R is the Rydberg constant and α is the fine structure constant, which is dimensionless.] However the Bohr model incorrectly assigns the angular momentum to $n\hbar$, rather than the correct value of $\hbar\sqrt{l(l+1)}$, so it is surprising that the Bohr model gets anywhere near the right value. The agreement in the case n = 2 is seen to be fortuitous because the only possible value of l is 1 and $n^2 = 2l(l+1)$. It is not hard to see that this case (n = 2, l = 1) is the only case where agreement can be expected.

4. Electromagnetic waves

Frequency, wavelength and speed.

4.1A $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = 254 \text{ nm}$, $\omega = 2\pi\nu \Rightarrow \nu = 1.18\times10^{15} \text{ Hz}$, $c = \lambda\nu = 3.00\times10^{8} \text{ m s}^{-1}$

the wave propagates with the speed of light in the $-x$ direction

4.2A $\lambda = c/\nu = 7.50\ \mu\text{m}$

Law of refraction, refractive index and speed of light.

4.3A (*a*) n = 1.360.

(*b*) $\lambda_{vac} = \frac{c_{vac}}{\nu} = \frac{c_{vac}}{c_1}\lambda_1 = n_1\lambda_1 = 892 \text{ nm}$

4.4A $\lambda_{vac} = n_1\lambda_1 = n_2\lambda_2 \Rightarrow \lambda_2 = \frac{n_1}{n_2}\lambda_1 = 435.6 \text{ nm}$

4.5A $\lambda_{vac} = n_1\lambda_1 = 656.45376 \text{ nm}$. Note that you must make this correction if you want precision greater than 3SF in the wavelength. This is important in the quantitative aspects of spectroscopy.

4.6B Draw a diagram and follow the refracted rays.

4.7B (a) $\theta_{w,c} = 48.61°$ (or 0.8483 radians).

(b) the diameter is 0.726 m.

4.8C This has not been covered in the lectures. Fermat's principle of least time says that the light ray from one point to another will follow the path that minimised the time taken. This is because waves on other paths will interfere destructively, but because this is a minimum there are many waves with similar paths and there is constructive interference.

Choose 2 points $(x_1, 0, z_1)$ in medium 1 with refractive index n_1 and $(x_2, 0, z_2)$ in medium 2 with refractive index n_2. The interface between the media is the plane $z = 0$, so that $z_1 > 0$ and $z_2 < 0$.

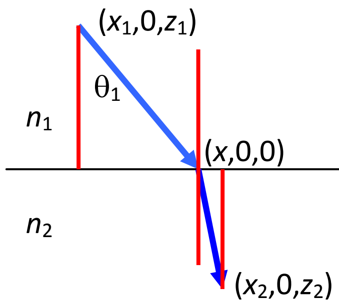
We wish to find the path between these points that minimises the time of travel. Suppose that this path cuts the interface at the point $(x, y, 0)$. Then the ray travels a distance $\sqrt{(x-x_1)^2 + y^2 + z_1^2}$ in medium 1 with speed c/n_1 and a distance $\sqrt{(x-x_2)^2 + y^2 + z_2^2}$ with speed c/n_2. The time taken is therefore $t(x,y) = \frac{n_1}{c}\sqrt{(x-x_1)^2 + y^2 + z_1^2} + \frac{n_2}{c}\sqrt{(x-x_2)^2 + y^2 + z_2^2}$, and we wish to find the point $(x, y, 0)$ that minimises this time.

$$\frac{\partial t}{\partial x} = \frac{n_1}{c}\frac{(x-x_1)}{\sqrt{(x-x_1)^2 + y^2 + z_1^2}} + \frac{n_2}{c}\frac{(x-x_2)}{\sqrt{(x-x_2)^2 + y^2 + z_2^2}} = 0$$

$$\frac{\partial t}{\partial y} = \frac{n_1}{c}\frac{y}{\sqrt{(x-x_1)^2 + y^2 + z_1^2}} + \frac{n_2}{c}\frac{y}{\sqrt{(x-x_2)^2 + y^2 + z_2^2}} = 0$$

In the second of these both terms have the same sign, the sign of y, and hence the only possible solution is $y = 0$, i.e. the three points lie in the same plane.

In the first of these the terms may only have opposite signs if x lies between x_1 and x_2. Drawing a diagram,

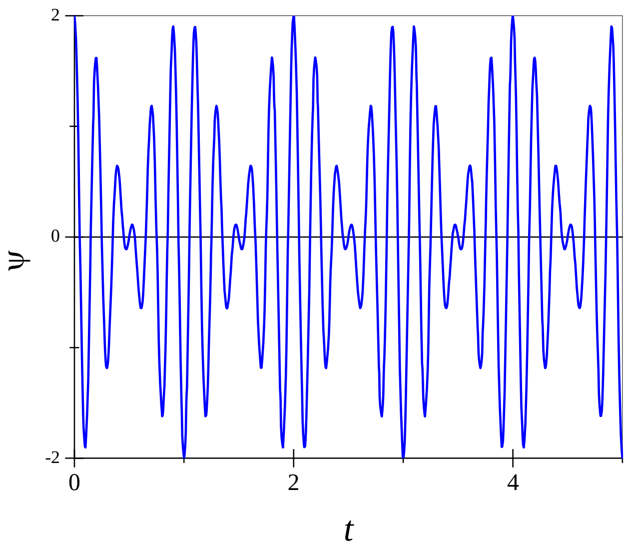


It is clear that $\frac{(x-x_1)}{\sqrt{(x-x_1)^2+z_1^2}}=\sin\theta_1$ and $\frac{(x_2-x)}{\sqrt{(x-x_2)^2+z_2^2}}=\sin\theta_2$, from which $n_1\sin\theta_1-n_2\sin\theta_2=0$, which is Snell's law.

Principle of superposition, interference.

4.9B $\psi=2A\cos(kx-\omega t+\pi/6)\cos(\pi/6)=\sqrt{3}A\cos(kx-\omega t+\pi/6)$

4.10A $\cos(\omega-\varepsilon)t+\cos(\omega+\varepsilon)t=2\cos\omega t\cos\varepsilon t$



the sketch has been done for ω = 10π and ε = π. Note that the beat frequency is double what you would expect: the peaks and troughs of the beating wave both appear as maxima in the envelope because the carrier wave passes through many cycles during each beat.

4.12B The distance from the centre of the slits to the screen at an angle θ is $D/\cos\theta$. If the screen is far away the two beams are almost parallel and the difference in path length between the light from one slit and the light from the other is $d\sin\theta$. Bright fringes correspond to constructive interference, i.e. to $d\sin\theta = n\lambda$.

The distance from the centre of the pattern is $x = D\tan\theta$, then $\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} = \frac{x}{\sqrt{D^2+x^2}}$ hence the *n*th bright fringe is at angle $n\lambda = d\sin\theta = \frac{xd}{\sqrt{D^2+x^2}}$, i.e. $x_n = \frac{\pm n\lambda D}{\sqrt{d^2 - n^2\lambda^2}}$.

If the angles are small it is acceptable to approximate $\sin\theta \approx \tan\theta$ and then $\sin\theta = \frac{x}{D}$, $n\lambda = \frac{xd}{D}$ and $x_n = \pm\frac{n\lambda D}{d}$

(*b*) Using $n\lambda = \frac{xd}{\sqrt{D^2+x^2}}$ we can deduce that $d = n\lambda\frac{\sqrt{D^2+x^2}}{x} \approx n\lambda\frac{D}{x}$. In the current instance the approximate form is good to within 0.05% and $d = 0.200\text{ mm}$.

4.13B $\psi_1 + \psi_2 = A\cos(kx - \omega t) + A\cos(kx + \omega t) = 2A\cos kx\cos\omega t$

The wave at a given point oscillates with angular frequency ω, but the peaks no longer travel, they occur when $kx = n\pi$, rather than when $kx = n\pi \pm \omega t$. This is a *standing wave*.